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ON FEEDBACK SYSTEMS POSSESSING INTEGRITY WITH RESPECT TO ACTUAT--ETC(U)
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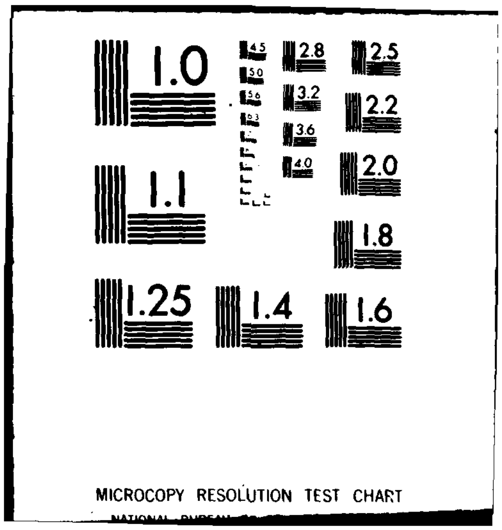
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On Feedback Systems Possessing Integrity With Respect to Actuator Outages*

LEVEL II

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ABSTRACT

For systems that are open-loop stable, there is a class of feedback controllers that have the property that the closed loop system is stable and remains stable in case actuator outages occur. Properties of a special subclass of these controllers are discussed.

*This work was supported by the Department of Energy under Contract DOE-ET-78-C-01-3391. The work was motivated by discussions with Mr. J.C. Doyle concerning the integrity property of controllers that have gains which are determined from Liapunov rather than Riccati equations.

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<u>INTRODUCTION</u>: Consider the linear controllable system

$$\dot{x} = Ax + Bu \tag{1}$$

where A is stable, i.e., the eigenvalues of A have negative real parts. The class of state feedback regulators of the form

$$u = -B^T Px \tag{2}$$

where P satisfies the Liapunov equation

$$PA + A^T P + Q = 0, \quad Q \geq 0 \tag{3}$$

with $(A, Q^{\frac{1}{2}})$ observable are of special interest because the closed loop systems with related regulators of the form

$$u = -LB^T Px, \quad L = L^T \geq 0 \tag{4}$$

are stable. Such regulators may be considered to possess integrity with respect to loss of imputs, that is, stability of the closed loop system is maintained when one or more inputs is set to zero. This situation of loss of inputs can be represented by an admissible L in (4). For example, the loss of the first input may be represented by taking L to be

$$L = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \tag{5}$$

The class of regulators defined by (2) and (3) is a subset of the class of optimal state feedback controllers. This fact and the proof of the stability for regulators of the form (4) are given below, followed by other results aimed at characterizing the class of regulators of interest.

<u>Optimality of Regulators given by (2), (3) and (4)</u>: The regulator given by (2) and (3) is the optimal regulator for (1) with respect to the performance index

$$J = \int_0^\infty \{x^T(Q + PBB^TP)x + u^Tu\}dt \qquad (6)$$

for P satisfying (3) and A being stable, $P \geq 0$, so that $Q + PBB^TP \geq 0$. The optimal regulator for (1) with respect to (6) is given by

$$u = -B^T\hat{P}x \qquad (7)$$

with $\hat{P}$ being the positive definite symmetric matrix solution to the Riccati equation

$$\hat{P}A + A^T\hat{P} + Q + PBB^TP - \hat{P}BB^T\hat{P} = 0 \qquad (8)$$

Thus, $\hat{P} = P$, and the controls given by (2) and (7) are the same. Now consider the control given by (3) and (4). If $L > 0$, this control is optimal with respect to the performance index.

$$J = \int_0^\infty \{x^T(Q + PBLB^TP)\ x + u^TL^{-1}u\}dt \qquad (9)$$

since the optimal control is given by

$$u = -(L^{-1})^{-1}\ B^TPx = -LB^TPx \qquad (10)$$

$$\begin{aligned} \text{where } 0 &= PA + A^TP + (Q + PBLB^TP) - PB(L^{-1})^{-1}\ B^TP \\ &= PA + A^TP + Q \end{aligned} \qquad (11)$$

If L is singular, the control $v = Tu$ is optimal with respect to

$$J = \int_0^\infty \{x^T(Q + PBLB^TP)\ x + v^Tv\}dt \qquad (12)$$

for the system

$$\dot{x} = Ax + BLT^T v \tag{13}$$

where $T^T T = L^{\dagger}$ (the pseudo-inverse of L) (14)

since $v = -(BLT^T)^T Px$ (15)

and
$$\begin{aligned} 0 &= PA + A^T P + (Q + PBLB^T P) - P(BLT^T)(BLT^T)^T P \\ &= PA + A^T P + Q + PBLB^T P - PBLT^T TLB^T P \\ &= PA + A^T P + Q \end{aligned} \tag{16}$$

Thus, the closed loop system given by (13) and (15) is stable, i.e.

$$\begin{aligned} \dot{x} &= (A - BLT^T(BLT^T)^T P)x \\ &= (A - BLT^T TLB^T P)x \\ &= [A + B(-LB^T P]x \end{aligned} \tag{17}$$

is stable. But (17) is the same closed loop system as that obtained using the control (4) in the system (1). This verifies the stability properties or integrity property of regulators of the special class of regulators described in the introduction.

Characterization of Regulators Defined by (2) and (3). A simple method of characterizing such regulators is to relate their closed loop parameters to the closed loop parameters of optimal linear regulators. This method can be applied to second order systems with a single input*, but appears to be intractable for general systems. For the simple example with

$$A = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad a > 0,\ b > 0 \tag{18}$$

*Single input systems also demonstrate that the set of controllers defined by (2) and (3) is a conservative estimate of the set of controllers with integrity because all stabilizing controllers, for single input systems possess integrity with respect to input outages.

the closed loop characteristic polynomial is $\hat{s}^2 + C_1\hat{s} + C_2$. Consider the two control laws, u_L defined by (2) and (3), and u_R defined by

$$u_R = -B^T P_R x \tag{19}$$

with $P_R > 0$ satisying the Riccati equation

$$P_R A + A^T P_R + Q_R - P_R B B^T P_R = 0 \tag{20}$$

The corresponding coefficients in the closed loop characteristic polynomial are:

$$C_{1L} = b + (q_{22}/2b) + (q_{11}/2ab),\ C_{2L} = a + (q_{11}/2a) \tag{21}$$

$$C_{1R} = [b^2 + q_{22R} + 2(\sqrt{a^2 + q_{11R}} - a]^{\frac{1}{2}},\ C_{2R} = \sqrt{a^2 + q_{11R}} \tag{22}$$

The sets of possible coefficients for these two control laws may be depicted in the (C_1, C_2) plane as shown in Figure 1. The set for u_L is a subset of the set for u_R. The lower boundaries of these two sets coincide (the line segment $C_2 = a$, $C_1 > b$). The upper boundary for the u_R set is the segment of the parabola $C_1^2 = b^2 + 2(C_2 - a)$ with $C_1 > b$. The upper boundary for the u_L set is the line segment, $b(C_1 - b) = C_2 - a$, $C_1 > b$ which is tangent to the upper boundary of the u_R set.

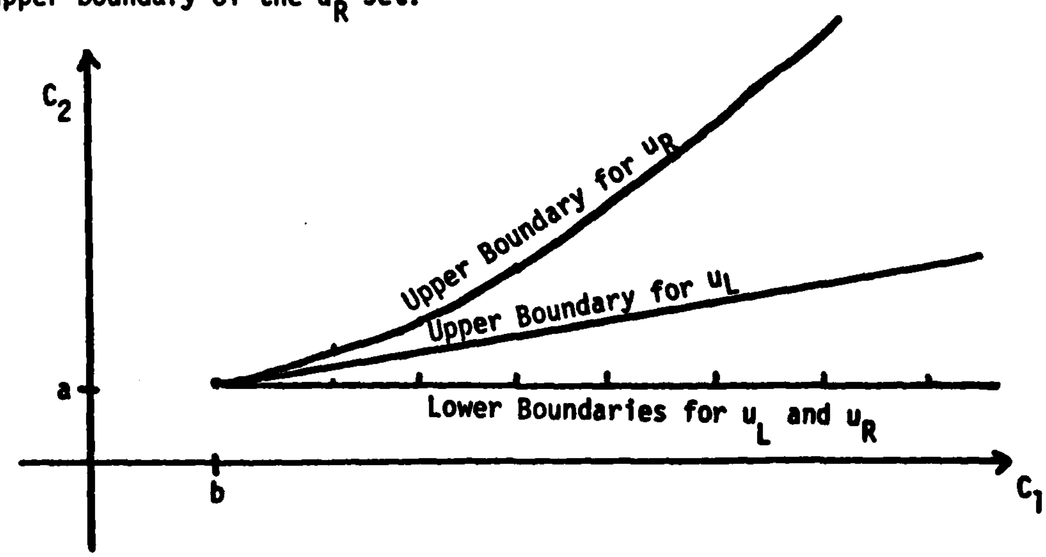

FIGURE 1. Possible Closed Loop Characteristic Polynomial Coefficients

The sets of possible closed loop roots may also be determined for this example. Figure 2 shows these sets for the case of a = b = 2. The negative real axis is contained in the possible sets for both u_L and u_R. The remaining set of possible roots for u_L is a small subset of the set of possible roots for u_R. Although these results appear to be impossible to generalize, let us note an interpretation of the sets of possible roots in this example that may be generalized. If we introduce

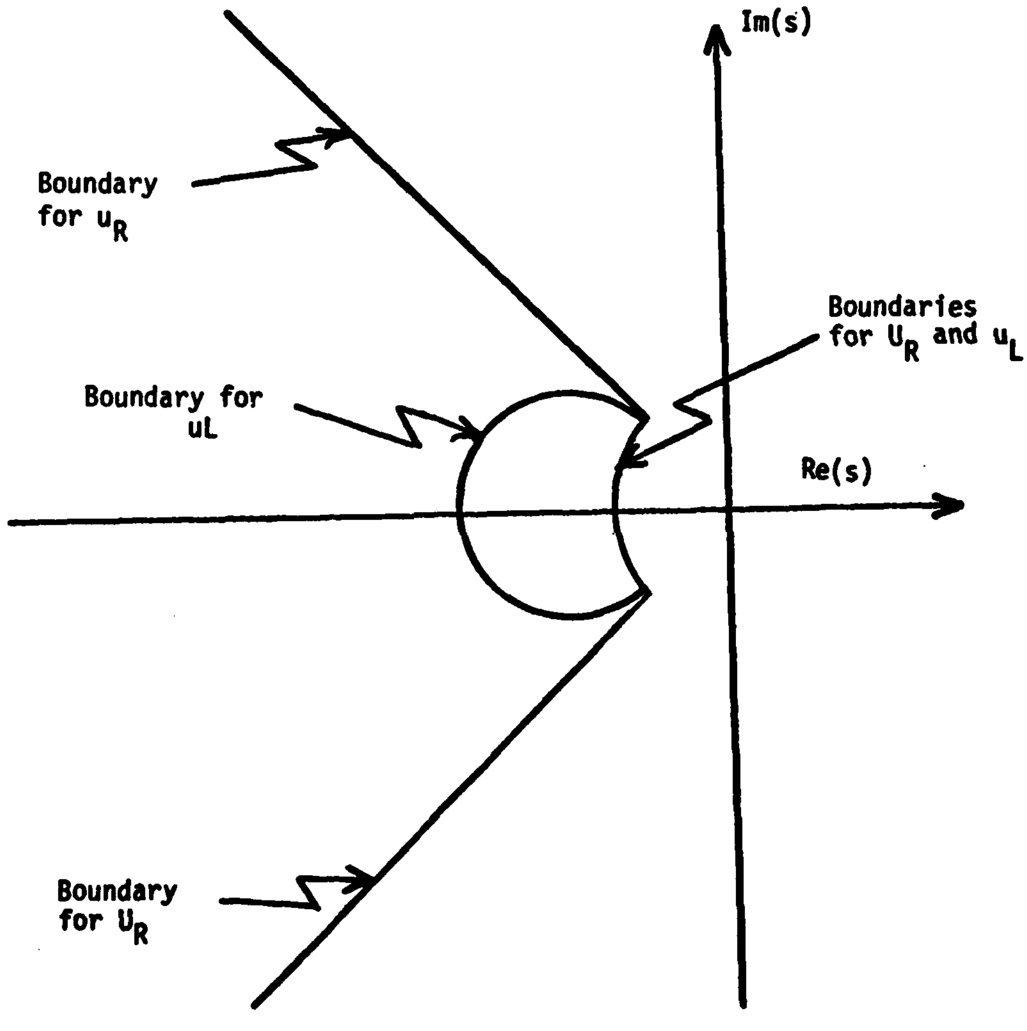

FIGURE 2. Possible Closed Loop Roots

a positive scalar parameter α in the matrices Q and Q_R, say $Q = \alpha\hat{Q}$ and $Q_R = \alpha\hat{Q}_R$ and consider the loci of closed loop roots as α tends to infinity.

these loci tend to zeros associated with $\hat{Q}$ and $\hat{Q}_R$. For this example there is at most one such zero and it must be real and negative. In the optimal case, the zero is a transmission zero associated with $\hat{Q}_R$ and its magnitude is arbitrary. In the case where u_L is obtained via (2) and (3) the magnitude of the zero associated with $\hat{Q}$ is restricted to be less than b.

In this example the root locus of interest is the locus of roots of the polynomial

$$p(s,\alpha) = s^2 + 2[b + (\alpha/2ab)(\hat{q}_{11} + a\hat{q}_{22})] + a + (\alpha/2a)\,\hat{q}_{11} \tag{23}$$

As α tends to infinity, the polynomial may be factored as

$$p(s,\alpha) = [s + b\hat{q}_{11}/(\hat{q}_{11}+a\hat{q}_{22}) + O(\alpha^{-1})][s + (\alpha/2ab)(\hat{q}_{11}+a\hat{q}_{22}) + O(1)] \tag{24}$$

so that one root tends to $-b\hat{q}_{11}/(\hat{q}_{11}+a\hat{q}_{22}) \geq -b$ and the other root tends to infinity.

Let us now return to the general case for the system (1) and introduce the parameter, α, into the control law, i.e.

$$u = -\alpha B^T P x \tag{25}$$

with P given by (3). Such a system may be characterized by asymptotic properties as the parameter, α, tends to infinity. The return difference matrix is

$$T(s) = I + \alpha B^T P(sI-A)^{-1}B \tag{26}$$

Algebraic manipulation leads to the result that

$$\begin{aligned} T^T(-s)\,T(s) = I &+ \alpha B^T(-sI-A^T)^{-1}Q(sI-A)^{-1}B \\ &+ \alpha^2\, B^T\,(-sI-A^T)^{-1}\,PBB^T P(sI-A)^{-1}B \end{aligned} \tag{27}$$

Let the dimension of x be n and the dimension of u be m. For simplicity, let us assume that $B^T PB$ has full rank. A generalization to the case in which the rank of $B^T PB$ is less than m is of interest but is also somewhat more complicated. If we let $s = \alpha\sigma$ in (27) and let α tend to infinity, we obtain

$$T^T(-\alpha\sigma)\ T(\alpha\sigma) \rightarrow I - \alpha^{-1}\ \sigma^{-2}\ B^TQB - \sigma^{-2}B^TPBB^TPB$$
$$\rightarrow \sigma^{-2}\ [\sigma^2 I - B^TPBB^TPB] \tag{28}$$

We also note that

$$\phi_c(-s)\phi_c(s) = \phi_0(-s)\ \phi_0(s)\ \det\ [T^T(-s)\ T(s)] \tag{29}$$

where $\phi_c(s)$ denotes the closed loop characteristic polynomial and $\phi_0(s)$ denotes the open loop characteristic polynomial. Since the closed loop system is stable for all $\alpha \geq 0$, we can deduce from (28) and (29) that m closed loop poles tend to infinity and are asymptotic to

$$\begin{aligned} s_i^\infty &= -\alpha\sqrt{\lambda_i(B^TPBB^TPB)},\quad i=1,2,\ldots,m \\ &= -\alpha\ \lambda_i(B^TPB),\ i=1,2,\ldots,m \end{aligned} \tag{30}$$

where $\lambda(A)$ denotes an eigenvalue of the matrix A. The remaining n-m closed loop poles approach finite values which are the left half plane images of the zeros of the determinant of $B^TP(sI-A)^{-1}B$. Denote these left half plane finite zeros by s_i^0, $i=1,2,\ldots,n-m$. The eigenvectors associated with the finite zeros are orthogonal to B^TP and are given by

$$X_i^0 = (s_i^0\ I-A)^{-1}\ B\mu_i^0\ ,\ i=1,2,\ldots,n-m \tag{31}$$

with the μ_i^0 determined by

$$B^TP(s_i^0\ I-A)^{-1}\ B\ \mu_i^0 = 0,\ i=1,2,\ldots,n-m \tag{32}$$

The eigenvectors associated with the asymptotically infinite eigenvalues are given by

$$X_i^\infty = B\ \mu_i^\infty,\quad i=1,2,\ldots,m \tag{33}$$

with μ_i^∞ determined by

$$\lambda_i(B^TPB)\mu_i^\infty = -B^TPB\ \mu_i^\infty,\ i=1,2,\ldots,m \tag{34}$$

These results are similar to those obtained for optimal regulators.*
For the optimal regulator, however, the asymptotic eigenvalues and eigenvectors are related to the weighting matrices Q_R and R_R. Here, unfortunately the relation is to P which is in turn related to Q, but a direct relationship to Q is not available.

In the special case for which $B^T = [0\ B_1^T]$ with B_1 an mxm nonsingular matrix, which would be the common representation for systems with actuator dynamics, we can proceed one step further. In this case, let $\nu_i^\infty = B_1\mu_i^\infty$. Then from (33)

$$X_i^\infty = \begin{bmatrix} 0 \\ B_1 \end{bmatrix} \mu_i^\infty = \begin{bmatrix} 0 \\ \nu_i^\infty \end{bmatrix},\ i=1,2,\ldots,m \tag{35}$$

and from (34)

$$\lambda_i\nu_i^\infty = -B_1(B^TPB)\ B_1^{-1}\ \nu_i^\infty = -B_1B_1^T\ P_4\ \nu_i^\infty,\ i=1,2,\ldots,m \tag{36}$$

where P_4 is the lower right mxm block of P. If we set $N = [\nu_1^\infty,\nu_2^\infty,\ldots,\nu_m^\infty]$ and $\Lambda = \text{diag}\ (\lambda_i^\infty)$, then (36) may be written as

$$N\Lambda = -B_1B_1^TP_4N \tag{37}$$

Thus,

$$P_4 = -(B_1B_1^T)^{-1}N\Lambda N^{-1} \tag{38}$$

and the fact that P_4 is symmetric implies constraints on N and Λ, which may be interpreted as constraints on actuator couplings and relative bandwidths.

*"Quadratic Weights for Asymptotic Regulator Properties", C.A. Harvey, G. Stein, IEEE Trans. on Auto. Control, Vol. AC-23, June 1978.

The fact that the finite zero, s_i^o, was constrained in the simple example is a property that is common to the general case. Adding $2s_i^oP$ to both sides of equation (3) and rearranging yields

$$P(s_i^oI-A) + (s_i^oI-A)^TP = Q + 2s_i^oP \tag{39}$$

Multiplying equation (39) on the right by $v_i^o = (s_i^oI-A)^{-1}\, B\mu_i^o$ and on the left by $(v_i^o)^T$ yields

$$2(\mu_i^o)^TB^TP\, v_i^o = (v_i^o)^T\,(Q + 2s_i^oP)v_i^o \tag{40}$$

But, $B^TP\, v_i^o = B^TP(s_i^oI-A)^{-1}\, B\mu_i^o = 0$ from (32), so that

$$-s_i^o = \frac{(v_i^o)^T\, Q\, v_i^o}{2(v_i^o)^T\, P\, v_i^o} \tag{41}$$

and it follows that

$$|s_i^o| \le \frac{\bar{\sigma}(Q)}{2\underline{\sigma}(P)} \tag{42}$$

where $\bar{\sigma}(Q)$ is the largest singular value of Q and $\underline{\sigma}(P)$ is the smallest singular value of P. Thus, (42), shows that the magnitudes of the finite zeros are bounded, but the bound involves P and Q. Since P is a function of A and Q, the bound is an implicit function of A and Q. Unfortunately, the explicit dependence is not evident.

An alternate characterization of controllers defined by (2) and (3) can be derived by considering the optimal controllers for (1) with respect to the performance index

$$J = \int_0^\infty (\beta x^TQx + u^Tu)dt \tag{43}$$

with β a small positive scalar parameter. In this case the optimal controller can be represented as

$$u = -B^T\left(\sum_{i=0}^{\infty} \beta^i P_i\right)x \tag{44}$$

where

$$\left(\sum_{i=0}^{\infty} \beta^i P_i\right) A + A^T\left(\sum_{i=0}^{\infty} \beta^i P_i\right) + \beta Q = \left(\sum_{i=0}^{\infty} \beta^i P_i\right) B B^T\left(\sum_{i=0}^{\infty} \beta^i P_i\right) \tag{45}$$

Equating terms of like powers of β in equation (45) yields

$$P_0A + A^TP_0 = P_0 B B^T P_0 \tag{45a}$$

$$P_1A + A^TP_1 + Q = P_0B B^TP_1 + P_1B B^TP_0 \tag{45b}$$

etc.

With A being a stable matrix, $P_0 = 0$, so that the right hand side of (45b) is zero. Thus, the equation for P_1 is the same as equation (3), and controllers defined by (2) and (3) may be viewed as multiples of first order approximation to optimal controllers, i.e.

$$u = -B^TP_1x = -B^T \lim_{\beta\to 0} \left[\frac{1}{\beta} \sum_{i=0}^{\infty} \beta^iP_i\right]x \tag{46}$$

Another way of describing this characterization is to consider the controller given by (2) and (3) with Q in (3) replaced by βQ. Then this controller is the first order approximation to the controller given by (44) as β tends to zero. This implies that the closed loop root loci (parameterized with β) associated with these controllers are tangent at $\beta = 0$ which corresponds to the open loop roots.